MENIIT
NEET | IIT-JEE | FOUNDATION
Corporate Office: 44-A/1, Kalu Sarai, New Delhi 110016 | Web: www.meniit.com

JEE MAINS-2013

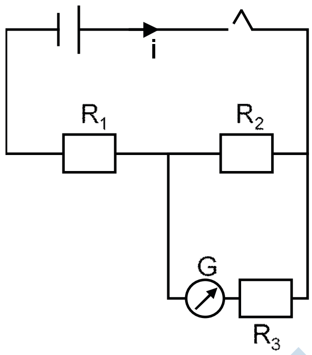
22-04-2013 (Online-2)

IMPORTANT INSTRUCTIONS

1. Immediately fill in the particulars on this page of the Test Booklet with **Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.**
2. The test is of **3** hours duration.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
5. Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

PART-A-PHYSICS

1. To find the resistance of a galvanometer by the half deflection method the following circuit is used with resistances $R_1 = 9970\ \Omega$, $R_2 = 30\Omega$ and $R_3 = 0$. The deflection in the galvanometer is d. With $R_3 = 107\ \Omega$ the deflection changed to $\frac{d}{2}$. The galvanometer resistance is approximately :



(A) $137\ \Omega$ (B) $107\ \Omega$ (C*) $77\ \Omega$ (D) $107/2\Omega$

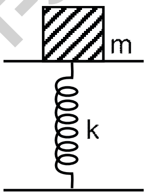
2. Given that 1 g of water in liquid phase has volume 1 cm^3 and in vapour phase 1671 cm^3 at atmospheric pressure and the latent heat of vaporization of water is 2256 J/g ; the change in the internal energy in Joules for 1 g of water at 373 K when it changes from liquid phase to vapour phase at the same temperature is :

(A*) 2089 (B) 2256 (C) 1 (D) 167

3. A mass m = 1.0 kg is put on a flat pan attached to a vertical spring fixed on the ground. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes simple harmonic motion. The spring constant is 500 N/m. What is the amplitude A of the motion, so that the mass m tends to get detached from the pan ? (Take g = 10 m/s^2).

The spring is stiff enough so that it does not get distorted during the motion.

(A) A = 2.0 cm
(B) A = 1.5 cm
(C*) A < 2.0 cm
(D) A > 2.0 cm



Ans. As F = –kx

4. The focal length of the objective and the eyepiece of a telescope are 50 cm and 5 cm respectively. If the telescope is focussed for distinct vision on a scale distant 2 m from its objective, then its magnifying power will be :

(A) – 8 (B*) – 2 (C) – 4 (D) + 8

Ans. Given : $f_0 = 50$ cm, $f_e = 5$ cm

d = 25 cm, $u_0 = -200$ cm

Magnification M = ?

As $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

$$\Rightarrow \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{50} - \frac{1}{200} = \frac{4-1}{200} = \frac{3}{200}$$

or $v_0 = \frac{200}{3}$ cm

Now $v_e = d = -25$ cm

From, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$-\frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e}$$

$$= \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$$

or, $v_e = \frac{-25}{6}$ cm

Magnification $M = M_0 \times M_e$

$$= \frac{v_0}{u_0} \times \frac{v_e}{u_e} = \frac{-200/3}{200} \times \frac{-25}{-25/6}$$

$$= -\frac{1}{3} \times 6 = -2$$

5. The half-life of a radioactive element A is the same as the mean-life of another radioactive element B. Initially both substances have the same number of atoms, then :

(A) A and B both decay at the same rate always

(B) A and B will decay at the same rate initially

(C) A will decay at a faster rate than B initially

(D*) B will decay at a faster rate than A initially

Ans. $(T_{1/2})_A = (t_{mean})_B$

$$\Rightarrow \frac{0.693}{\lambda_A} = \frac{1}{\lambda_B} \Rightarrow \lambda_A = 0.693\lambda_B$$

or $\lambda_A < \lambda_B$

Also rate of decay = λN

Initially number of atoms (N) of both are equal but since $\lambda_B > \lambda_A$, therefore B will decay at a faster rate than A.

6. An ideal gas at atmospheric pressure is adiabatically compressed so that its density becomes 32 times of its initial value. If the final pressure of gas is 128 atmospheres, the value of 'γ' of the gas is :

(A) 1.6 (B) 1.5 (C) 1.3 (D*) 1.4

Ans. Volume of the gas

$v = \frac{m}{d}$ and

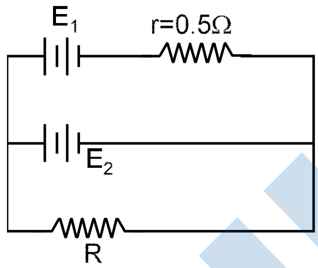
Using PV^{γ} = constant

$$\frac{P'}{P} = \frac{V}{V'} = \left(\frac{d'}{d}\right)^{\gamma}$$

or $128 = (32)^{\gamma}$

$$\therefore \gamma = \frac{7}{5} = 1.4$$

7. A dc source of emf E_1 = 100 V and internal resistance r = 0.5 Ω, a storage battery of emf E_2 = 90 V and an external resistance R are connected as shown in figure. For what value of R no current will pass through the battery ?



(A) 5.5 Ω (B*) 4.5 Ω (C) 3.5 Ω (D) 2.5 Ω

Ans. $\frac{100}{R+r} = \frac{90}{R}$

$\Rightarrow \frac{R+r}{R} = \frac{10}{9}$

$\Rightarrow 1 + \frac{0.5}{R} = \frac{10}{9}$

$\Rightarrow \frac{0.5}{R} = \frac{1}{9}$

$\therefore$ R = 4.5 Ω

8. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statement.

Statement 1 : In Young's double slit experiment, the number of fringes observed in the field of view is small with longer wave length of light and is large with shorter wave length of light.

Statement 2 : In the double slit experiment the fringe width depends directly on the wave length of light

(A) Statement 1 is false and statement 2

(B*) Statement 1 and statement 2 both are true. But statement 2 is not the correct explanation of statement 1

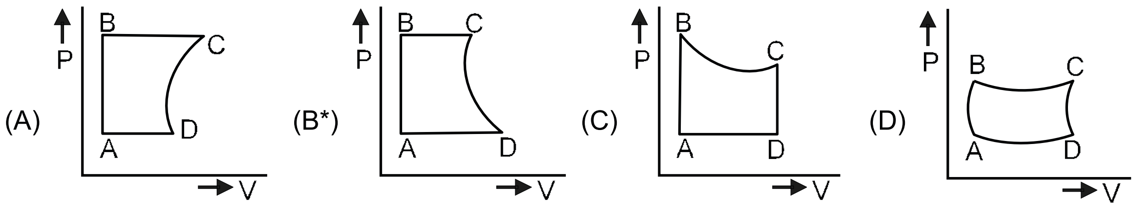
(C) Statement 1 is true and Statement 2

(D) Statement 1 and Statement 2 both are true. Statement 2 is the correct explanation of Statement 1.

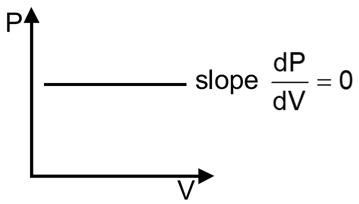
Ans. Fringe width $B = \frac{D}{d}\lambda$

And number of fringes observed in the field of view is obtained by $\frac{d}{\lambda}$

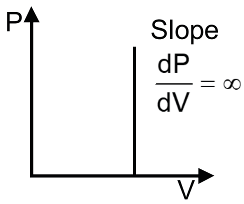
9. A certain amount of gas is taken through a cyclic process (A B C D A) that has two isobars, one isochore and one isothermal. The cycle can be represented on a P-V indicator diagram as:



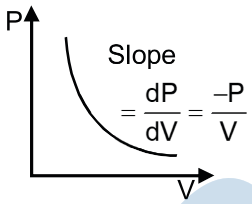
Ans. P-V indicator diagram for isobaric



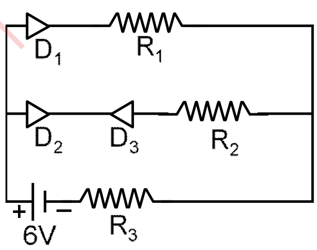
P-V indicator diagram for isochoric process



P-V indicator diagram for isothermal process



10. Figure shows a circuit in which three identical diodes are used. Each diode has forward resistance of 20 Ω and infinite backward resistance. Resistors $R_1 = R_2 = R_3 = 50\ \Omega$. Battery voltage is 6 V. The current through R_3 is :



(A) 60 mA

(B) 100 mA

(C*) 50 mA

(D) 25 mA

Ans. Here, diodes D_1 and D_2 are forward biased and D_3 is reverse biased.

$$i = \frac{V}{R'} = \frac{6}{120} = \frac{1}{20} A = 50\ mA$$

11. To establish an instantaneous current of 2 A through a 1 μF capacitor; the potential difference across the capacitor plates should be changed at the rate of :

(A) 4×10^4 V/s (B) 2×10^4 V/s (C) 4×10^6 V/s (D*) 2×10^6 V/s

Ans. As, $C = \frac{Q}{V} = \frac{It}{V}$

$\Rightarrow \frac{V}{t} = \frac{1}{C} = \frac{2}{1 \times 10^{-6}}$

$= 2 \times 10^6$ V/s

12. The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is :

(mass of the moon = 7.36×10^{22} kg, radius of the moon's orbit = 3.8×10^8 m).

(A*) $6.73 \times 10^{-5} \frac{m}{s^2}$ (B) $6.73 \times 10^{-4} \frac{m}{s^2}$

(C) $6.73 \times 10^{-3} \frac{m}{s^2}$ (D) $6.73 \times 10^{-2} \frac{m}{s^2}$

13. A current i is flowing in a straight conductor of length L. The magnetic induction at a point on its axis at a distance L/4 from its centre will be :

(A) $\frac{4\mu_0 i}{\sqrt{5}\pi L}$ (B) $\frac{\mu_0 i}{\sqrt{2}L}$ (C) $\frac{\mu_0 i}{2\pi L}$ (D*) Zero

Ans. Magnetic field at any point lies on axial position of current carrying conductor B = 0

14. A body starts from rest on a long inclined plane of slope 45°. The coefficient of friction between the body and the plane varies as $\mu = 0.3$ x, where x is distance travelled down the plane. The body will have maximum speed (for g = 10 m/s^2) when x =

(A) 9.8 m (B) 27 m (C*) 3.33 m (D) 12 m

Ans. When the body has maximum speed then

$\mu = 0.3x = \tan 45°$

$\therefore$ x = 3.33 m

15. The dimensions of angular momentum, latent heat and capacitance are, respectively.

(A*) ML^2T^{-1}, L^2T^{-2}, $M^{-1}L^{-2}T^4 A^2$ (B) $ML^2 T^{-2}$, L^2T2, $M^{-1}L^{-2}T^4A^2$

(C) ML^2T^{-1}, L^2T^{-2}, ML^2TA^2 (D) $ML^2T^1 A^2$, L^2T^{-2}, $M^{-1}L^{-2}T^2$

Ans. Angular momentum = m × v × r = ML^2T^{-1}

Latent heat $L = \frac{Q}{m} = \frac{ML^2T^{-2}}{M} = L^2T^{-2}$

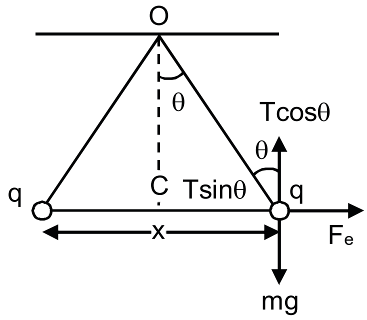
Capacitance $C = \frac{\text{Charge}}{\text{P.d.}} = M^{-1}L^{-2}T^4A^2$

16. Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating massless strings of equal lengths. They come to equilibrium with each string making angle θ from the vertical. If the mass of each charge is m, then the electrostatic potential at the centre of line joining them will be $\left(\frac{1}{4\pi \epsilon_0} = k\right)$:

(A) $\sqrt{k\,mg\tan\theta}$ (B) $\sqrt{k\,mg/\tan\theta}$ (C) $2\sqrt{k\,mg\tan\theta}$ (D*) $4\sqrt{k\,mg/\tan\theta}$

Ans. In equilibrium, F = T sin θ

mg = T cos θ



$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi \in_0 x^2 \times mg}$$

$$\therefore x = \sqrt{\frac{q^2}{4\pi \in_0 \tan\theta\, mg}}$$

Electric potential at the centre or the line

$$V = \frac{kq}{x/2} + \frac{kq}{x/2} = 4\sqrt{kmg/\tan\theta}$$

17. A plane electromagnetic wave in a non-magnetic dielectric medium is given by $\vec{E} = \vec{E}_0(4\times10^{-7}x - 50t)$ with distance being in meter and time in seconds. The dielectric constant of the medium is :

(A) 4.8 (B) 8.2 (C*) 5.8 (D) 2.4

18. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement 1 : Short wave transmission is achieved due to the total internal reflection of the e-m wave from an appropriate height in the ionosphere.

Statement 2 : Refractive index of a plasma is independent of the frequency of e-m waves.

(A) Statement 1 is false and statement 2 true

(B) Statement 1 and statement 2 both are true. But statement 2 is not the correct explanation of statement 1

(C*) Statement 1 is true and Statement 2 false

(D) Statement 1 and Statement 2 both are true. Statement 2 is the correct explanation of Statement 1.

Ans. Effective refractive index of the ionosphere

$$n_{eff} = n_0\left[1 - \frac{80.5N}{f^2}\right]^{1/2}$$

Where f is the frequency of em waves.

19. A uniform wire (Young's modulus 2×10^{11} Nm^{-2}) is subjected to longitudinal tensile stress of 5×10^7 Nm^{-2}. If the overall volume change in the wire is 0.02%, the fractional decrease in the radius of the wire is close to :

(A*) 0.25×10^{-4} (B) 1.0×10^{-4} (C) 5×10^{-4} (D) 1.5×10^{-4}

Ans. Given, y = 2×10^{11} Nm^{-2}

$$\text{Stress}\left(\frac{F}{A}\right) = 5\times10^7 Nm^{-2}$$

$\Delta V = 0.02\% = 2 \times 10^{-4}$ m^3

$$\frac{\Delta r}{r} = ?$$

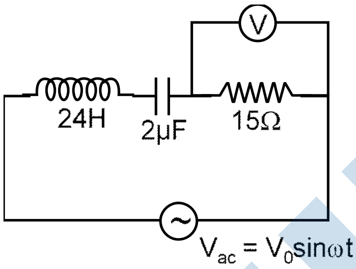
$$\gamma = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain}\left(\frac{\Delta\ell}{\ell_0}\right) = \frac{\gamma}{\text{stress}} \quad \ldots\ldots \text{(i)}$$

$$\Delta V = 2\pi r\ell_0\,\Delta r - \pi r^2\,\Delta\ell \quad \ldots\ldots \text{(ii)}$$

From eqns (i) and (ii) putting the value of $\Delta\ell$, ℓ_0 and ΔV and solving we gat

$$\frac{\Delta r}{r} = 0.25 \times 10^{-4}$$

20. An LCR circuit as shown in the figure is connected to a voltage source V_{ac} whose frequency can be varied. The frequency, at which the voltage across the resistor is maximum, is :

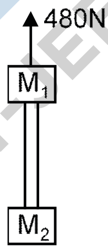


(A*) 23 Hz (B) 345 Hz (C) 143 Hz (D) 902 Hz

Ans. Frequency $f = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.4\sqrt{24 \times 2 \times 10^{-6}}} = 23 \text{ Hz}$$

21. Two blocks of mass M_1 = 20 kg and M_2 = 12 kg are connected by a metal rod of mass 8 kg. The system is pulled vertically up by applying a force of 480 N as shown. The tension at the mid-point of the rod is :



(A) 240 N (B) 96 N (C*) 192 N (D) 144 N

Ans. Acceleration produced in upward direction

$$a = \frac{F}{M_1 + M_2 + \text{Mass of metal rod}}$$

$$= \frac{480}{20 + 12 + 8} = 12\text{ms}^{-2}$$

Tension at the mid point

$$T = \left(M_2 + \frac{\text{Mass of rod}}{2}\right)a$$

$$= (12 + 4) \times 12 = 192\text{N}$$

22. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is :

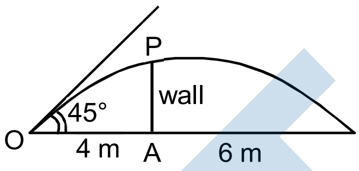
(A) 4.4 m (B*) 2.4 m (C) 3.6 m (D) 1.6 m

Ans. As ball is projected at an angle 45° to the horizontal therefore Range = 4H

or $10 = 4H \Rightarrow H = \frac{10}{4} = 2.5$ m

($\because$ Range = 4 m + 6 m = 10 m)

Maximum height, $H = \frac{u^2 \sin^2\theta}{2g}$



$\therefore u^2 = \frac{H \times 2g}{\sin^2\theta} = \frac{2.5 \times 2 \times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$

or, $u = \sqrt{100} = 10\ \text{ms}^{-1}$

Height of wall PA

$= OA\ \tan\theta - \frac{1}{2}\frac{g(OA)^2}{u^2\cos^2\theta}$

$= 4 - \frac{1}{2} \times \frac{10 \times 16}{10 \times 10 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = 2.4$ m

23. Orbits of a particle moving in a circle are such that the perimeter of the orbit equals an integer number of de - Broglie wavelengths of the particle. For a charged particle moving in a plane perpendicular to a magnetic field, the radius of the n^{th} orbital will therefore be proportional to :

(A) n^2 (B*) $n^{1/2}$ (C) $n^{1/4}$ (D) n

Ans. According to the question,

$2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$

or $mvr = \frac{nh}{2\pi}$ or $mv = \frac{nh}{2\pi r}$

$F = qv_B = \frac{mv^2}{r}$

or, $q_B = \frac{mv}{r} = \frac{nh}{2\pi r.r}$

or, $r^2 = \frac{nh}{2\pi qB}$

or, $r = \sqrt{\frac{nh}{2\pi qB}}$

i.e., $r \propto n^{1/2}$

24. Air of density 1.2 kg m^{-3} is blowing across the horizontal wings of an aeroplane in such a way that its speeds above and below the wings are 150 ms^{-1} and 100 ms^{-1}, respectively. The pressure difference between the upper and lower sides of the wings, is :

(A) 180 Nm^{-2} (B*) 7500 Nm^{-2} (C) 12500 Nm^{-2} (D) 60 Nm^{-2}

Ans. Pressure difference

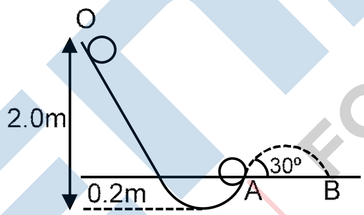
$$p_2 - p_1 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$= \frac{1}{2} \times 1.2\left((150)^2 - (100)^2\right)$$

$$= \frac{1}{2} \times 1.2(22500 - 10000)$$

= 7500 Nm^{-2}

25. A tennis ball (treated as hollow spherical shell) starting from O rolls down a hill. At point A the ball becomes air borne leaving at an angle of 30° with the horizontal. The ball strikes the ground at B. What is the value of the distance AB ? (Moment of inertia of a spherical shell of mass m and radius R about its diameter $= \frac{2}{3}mR^2$)



(A*) 1.87 m (B) 1.77 m (C) 1.57 m (D) 2.08 m

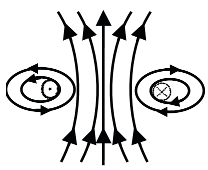
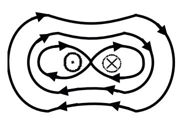
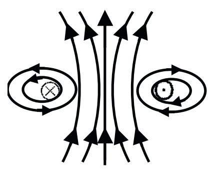
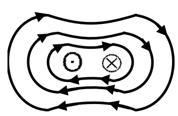
Ans. Velocity of the tennis ball on the surface of the earth or ground

$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$ (Where k = radius of gyration of spherical shell = $\sqrt{\frac{2}{3}}R$)

Horizontal range $AB = \frac{v^2 \sin 2\theta}{g}$

$$= \frac{\left(\sqrt{\frac{2gh}{1 + k^2/R^2}}\right)\sin(2 \times 30°)}{g}$$

26. Choose the correct sketch of the magnetic field lines of a circular current loop shown by the dot ⊙ and the cross ⊗.

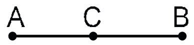
(A*)  (B)  (C)  (D) 

Ans. If magnetic field is perpendicular and into the plane of the paper, it is represented by cross ⊗ and if the direction of the magnetic field is perpendicular out of the plane of the paper it is represented by dot ʘ.

27. In a series L-C-R circuit, $C = 10^{-11}$ Farad, $L = 10^{-5}$ Henry and R = 100 Ohm, when a constant D.C voltage E is applied to the circuit, the capacitor acquires a charge 10^{-9}C. The D.C. source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant D.C. voltage E. At resonance the peak value of the charge acquired by the capacitor will be :

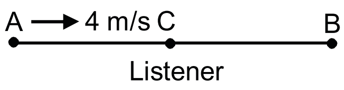
(A) 10^{-10} C (B*) 10^{-8} C (C) 10^{-15} C (D) 10^{-6} C

28. A and B are two sources generating sound waves. A listener is situated at C. The frequency of the source at A is 500 Hz. A, now, moves towards C with a speed 4 m/s. The number of beats heard at C is 6. When A moves away from C with speed 4 m/s, the number of beats heard at C is 18. The speed of sound is 340 m/s. The frequency of the source at B is :



(A) 506 Hz (B) 500 Hz (C*) 512 Hz (D) 494 Hz

Ans. f = 500 Hz



Cas 1 : When source is moving towards stationary listener

apparent frequency $\eta' = \eta\left(\frac{v}{v - v_s}\right)$

$= 500\left(\frac{340}{336}\right) = 506$ Hz

Cas 2 : When source is moving away from the stationary listener

$\eta'' = \eta\left(\frac{v}{v + v_s}\right) = 500\left(\frac{340}{344}\right) = 494\,\text{Hz}$

In case I number of bats heard is 6 and in case 2 number of bats heard is 18 therefore frequency of the source at B = 512 Hz

29. The image of an illuminated square is obtained on a screen with the help of a converging lens. The distance of the square from the lens is 40 cm. The area of the image is 9 times that of the square. The focal length of the lens is :

(A) 60 cm (B*) 30 cm (C) 36 cm (D) 27 cm

Ans. If side of object square = ℓ

and side of image square = ℓ'

From question, $\frac{\ell'^2}{\ell} = 9$

or $\frac{\ell'}{\ell} = 3$

u = – 40 cm

v = 3 × 40 = 120 cm

f = ?

Form formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{120} - \frac{1}{-40} = \frac{1}{f}$

or, $\frac{1}{f} = \frac{1}{120} - + \frac{1}{40} = \frac{1+3}{120} \therefore f = 30$ cm

30. A point charge of magnitude +100μC is fixed at (0,0,0). An isolated uncharged spherical conductor, is fixed with its center at (4, 0, 0). The potential and the induced electric field at the centre of the sphere is:

(A) 1.8×10^5 V and – 5.625×10^6 V/m

(B*) 2.25×10^5 V and – 5.625×10^6 V/m

(C) 2.25×10^5 V and 0 V/m

(D) 0 V and 0 V / m

Ans. $q = 1\mu C = 1 \times 10^{-6}$ C

$r = 4$ cm $= 4 \times 10^{-2}$ m

Potential $V = \frac{kq}{r}$

$= \frac{9 \times 10^9 \times 10^{-6}}{4 \times 10^{-2}}$

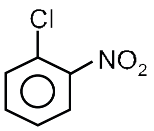
$= 2.25 \times 10^5$ V.

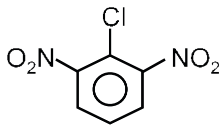
Induced electric field $E = -\frac{kq}{r^2}$

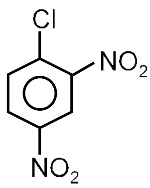
$= \frac{9 \times 10^9 \times 1 \times 10^{-6}}{16 \times 10^{-4}} = 5.625 \times 10^6$ V/ m

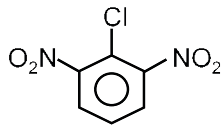
PART-B-CHEMISTRY

31. A major component of Borsch reagent is obtained by reacting hydrazine hydrate with which of the following ?

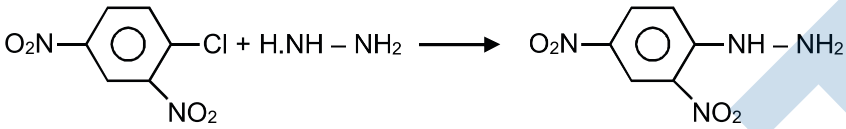
(A) 

(B) 

(C*) 

(D) 

Ans. The major component of Borsch reagent is 2,4- dinitrophenyl hydrazine which can b obtained by reaction of 2,4dinitrochloro benzene and hydrazine.



2,4-dinitrophenyl hydrazine

32. In Williamson synthesis of mixed ether having a primary and a tertiary alkyl group if tertiary halide is used, then :

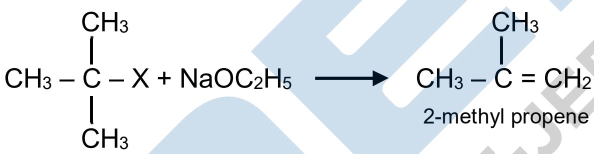
(A*) Alkene will be the main product

(B) Simple ether will instead of mixed ether

(C) Expected mixed ether will be formed

(D) Rate of reaction will be slow due to show cleavage of carbon-halogen bond

Ans. Th tertiary alkyl halide undergo elimination reaction to give alkenes



2-methyl propene

33. The Wurtz-Fittig reaction involves condensation of :

(A) Two molecules of aralkyl-halides

(B) one molecule of each aryl-halide and phenol

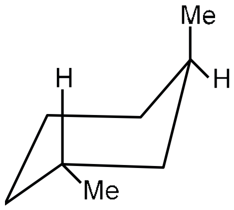
(C) two molecules of aryl halides

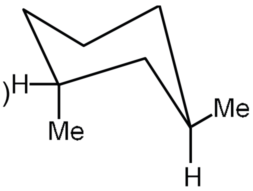
(D*) one molecule of each of aryl-halide and alkyl-halide

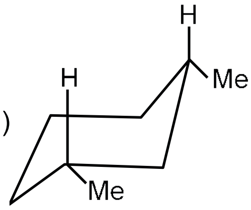
Ans. Reaction between alkyl halide, aryl halide and sodium in presence of ether is known as Wurtz fitting reaction

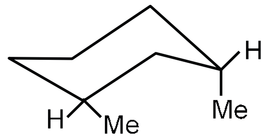
$$C_6H_5Cl + 2Na + ClCH_3 \longrightarrow \underset{\text{Toluene}}{C_6H_5CH_3} + 2NaCl$$

34. Arrange in the correct order of stability (decreasing order) for the following molecules:

(I) 

(II) 

(III) 

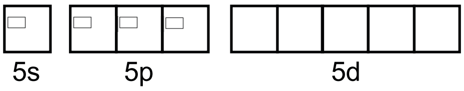
(IV) 

(A) III > I ≈ II > IV (B) IV > III > II ≈ I (C*) I > II ≈ III > IV (D) I > II > III > IV

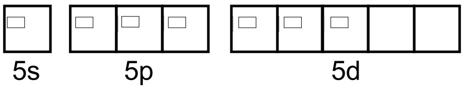
35. XeO_4 molecule is tetrahedral having :

(A*) Four pπ–dπ bonds (B) One pπ–dπ bond (C) Three pπ–dπ bonds (D) Two pπ-dπ bonds

Ans. Xenon undergo sp^3 hybridization.

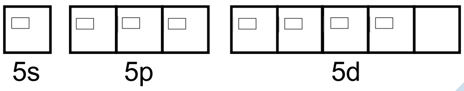


(ground state)



(Third excited state)

In the fourth excited state xenon atom, has 8 unpaired electrons



One s and three p orbital undergo sp^3 hybridization. Four sp^3 hybrid orbitals form four σ bonds with oxygen atoms. They are $\sigma sp^3 - p$. Four pπ – dπ bonds are also formed with oxygen atoms by the unpaired electrons.

36. Oxidation state of sulphur in anions SO_3^{2-}, $S_2O_4^{2-}$ and $S_2O_6^{2-}$ increases in the orders :

(A) $S_2O_6^{2-} < S_2O_4^{2-} < SO_3^{2-}$ (B) $SO_3^{2-} < S_2O_4^{2-} < S_2O_6^{2-}$

(C*) $S_2O_4^{2-} < SO_3^{2-} < S_2O_6^{2-}$ (D) $S_2O_4^{2-} < S_2O_6^{2-} < SO_3^{2-}$

Ans. In SO_3^{--}

$x + 3(-2) = -2;\ x = +4$

In $S_2O_4^{--}$

$2x + 4(-2) = -2$

$2x - 8 = -2$

$2x = 6;\ x = +3$

In $S_2O_6^{2-}$

$2x + 6(-2) = -2$

$2x = 10;\ x = +5$

Hence the correct order is

$S_2O_4^{--} < SO_3^{--} < S_2O_6^{--}$

37. A molecule M associates in a given solvent according to the equations M ⇌ $(M)_n$. For a certain concentration of M, the van't Hoff factor was found to be 0.9 and the fraction of associated molecules was 0.2. The value of n is :

(A) 4 (B) 5 (C*) 2 (D) 3

Ans. van't Hoff factor(i) and the degree of association are related as below:

$$i = 1 - \alpha\left(1 - \frac{1}{n}\right)$$

$$0.9 = 1 - 0.2\left(1 - \frac{1}{n}\right)$$

On solving,

$$\left(1 - \frac{1}{n}\right) = \frac{1}{2}$$

$$\frac{1}{n} = 1 - \frac{1}{2} = \frac{1}{2}$$

$\therefore$ n = 2

38. The solubility order for alkali metal fluoride in water is :

(A) RbF < KF < NaF < LiF (B) LiF < RbF < KF < NaF

(C) LiF > NaF > KF > RbF (D*) LiF < NaF < KF < RbF

Ans. Higher the lattice enthalpy lower will be solubility i.e.,

$$\text{lattice enthalpy} \propto \frac{1}{\text{Solubility}}$$

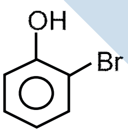
Since the lattice enthalpy of alkali metals follow the order

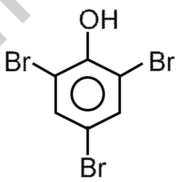
Li > Na > K > Rb

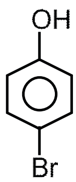
Hence the correct order of solubility is

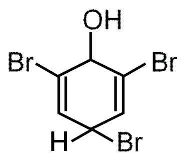
LiF < NaF < KF < RbF

39. What is the structure of the major product when phenol is treated with bromine water ?

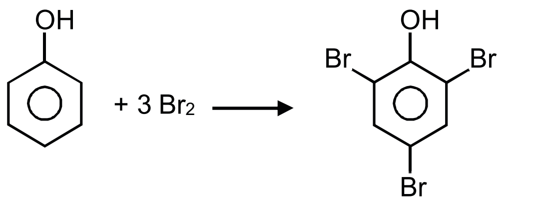
(A) 

(B*) 

(C) 

(D) 

Ans. Phenol has activation (electron releasing) –OH group and bromine water supplies Br^+ ion easily, hence under such conditions reaction does not stop at mono bromo or dibromo stage but a fully brominated (2,4,6-tribromophenol) compound is the final product.



2,4,6-Tribromophenol

40. Which of the following statements is incorrect?

(A) Fe^{3+} ion gives blood red colour with SCN^- ion

(B*) Fe^{2+} ion also gives blood red colour with SCN^- ion.

(C) On passing H_2S into Na_2ZnO_2 solution, a white ppt of ZnCl is formed.

(D) Cupic ion react with excess of ammonia soln to give deep blue colour of $[Cu(NH_3)_4]^{2+}$ ion

Ans. Only Fe^{3+} ions give bloodred colouration with SCN^-m ions.

$$Fe^{3+} + SCN^- \longrightarrow \underset{\text{(dark red)}}{[Fe(SCN)]^{2+}}$$

41. Values of dissociation constant, K_a are given as follows :

Acid	**K_a**
HCN	6.2×10^{-10}
HF	7.2×10^{-4}
HNO_2	4.0×10^{-4}

Correct order of increasing base strength of the base CN^-, F^- and NO_2^- will be :

(A*) $F^- < NO_2^- < CN^-$ (B) $NO_2^- < CN^- < F^-$ (C) $F^- < CN^- < NO_2^-$ (D) $NO_2^- < F^- < CN^-$

Ans. higher the value of K_a lower will be the value of pK_a i.e., higher will be the acidic nature. Further since CN^-, F^- andNo_2^- are conjugated base of the acids HCN, HF and HNO_2 respectively hence the correct order of base strength will be

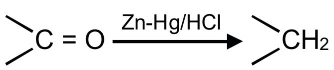
$F^- < NO_2^- < CN^-$

($\because$ stronger the acid weaker will be its conjugate base)

42. Clemmensen reduction of a ketone is carried out in the presence of :

(A*) Zn–Hg with HCl (B) H_2 with Pt as catalyst

(C) Glycol with KOH (D) $LiAlH_4$

Ans. 

Ketone

43. Which of the following would not give 2–phenylbutane as the major product in a Friedel-Crafts alkylation reaction ?

(A*) Butanoyl chloride + $AlCl_3$ then Zn, HCl (B) 2–butanol + H_2SO_4

(C) 1–butene + HF (D) Butyl chloride + $AlCl_3$

Ans. The Friedal-crafts alkylation reaction will give propyl phenyl ketone which further on Clemmenson's reduction will give butyl benzene

$$C_6H_6 + CH_3CH_2CH_2COCl \xrightarrow{AlCl_3} C_6H_5COCH_2CH_2CH_3 \xrightarrow{Zn-Hg/HCl} \underset{\text{Butyl benzene}}{C_6H_5CH_2CH_2CH_2CH_3}$$

44. Which one of the following arrangements represents the correct order of solubilities of sparingly soluble salts $Hg_2Cl_2, Cr_2(SO_4)_3$, $BaSO_4$ and $CrCl_3$ respectively ?

(A) $BaSO_4 > Hg_2Cl_2 > Cr_2(sO_4)_3 > CrCl_3$
(B*) $BaSO_4 > Hg_2Cl_2 > CrCl_3 > Cr_2(sO_4)_3$
(C) $Hg_2Cl_2 > CrCl_3 > BaSO_4 > Cr_2(sO_4)_3$
(D) $Hg_2Cl_2 > BaSO_4 > CrCl_3 > Cr_2(sO_4)_3$

Ans. $Cr_2(SO_4)_3 \rightleftharpoons \underset{2s}{2Cr^{+++}} + \underset{3s}{3SO_4^-}$

$K_{sp} = (2s)^2(3s)^2 = 4s^2 \times 27s^3 = 108s^5$

$$s = \left(\frac{K_{sp}}{108}\right)^{1/5}$$

$Hg_2Cl_2 \rightleftharpoons \underset{2s}{2Hg^{2+}} + \underset{2s}{2Cl^-}$

$K_{sp} = (2s)^2 \times (2s)^2 = 16s^4$

$$s = \left(\frac{K_{sp}}{16}\right)^{1/4}$$

$BaSO_4 \rightleftharpoons \underset{s}{Ba^{++}} + \underset{s}{SO_4^-}$

$K_{sp} = s^2$

$$s = \sqrt{K_{sp}}$$

$CrCl_3 \rightleftharpoons \underset{s}{Cr^{3+}} + \underset{3s}{3Cl^-}$

$K_{sp} = s \times (3s)^2 = 27\ s^4$

$$s = \left(\frac{K_{sp}}{27}\right)^{1/4}$$

Hence the correct order of solubilities of salts is

$$\sqrt{K_{sp}} > \left(\frac{K_{sp}}{16}\right)^{1/4} > \left(\frac{K_{sp}}{27}\right)^{1/4} > \left(\frac{K_{sp}}{108}\right)^{1/5}$$

45. Which of the following is diamagnetic ?

(A*) $[Co(ox)]^{3-}$ (B) $[FeF_6]^{3-}$ (C) $[Co(F_6)]^{3-}$ (D) $[Fe(CN)_6]^{3-}$

Ans. $[Fe(CN)_6]^{3-}$ has magnetic moment of a single unpaired electron whereas $[FeF_6]^{3-}$ has a magnetic moment of five unpaired electrons. $[CoF_6]^{3-}$ is paramagnetic with four unpaired electrons while $[Co(C_2O_4)_3]^3$ is diamagnetic. This anomaly is explained by valence bond theory in terms of formation of inner and outer orbital coordination entities. $[Co(C_2O_4)_3]^{3-}$ is an inner orbital complexes having d^2sp^3 hybridization.

46. Bond order normally gives idea of stability of a molecular species. All the molecules viz. H_2, Li_2 and B_2 have the same bond order yet they are not equally stable. Their stability order is :

(A) $Li_2 > H_2 > B_2$ (B) $Li_2 > B_2 > H_2$ (C) $H_2 > B_2 > Li_2$ (D) $B_2 > H_2 > Li_2$

Ans. None of the given option is correct.

The molecular orbital configuration of the given molecules is

$H_2 = \sigma 1s^2$ (no electron anti-bonding)

$Li_2 = \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2$ (two anti-bonding electron)

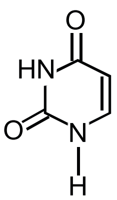
$B_2 = \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \{\pi 2p_y^1 = \pi 2p_z^1\}$ (4 anti-bonding electron)

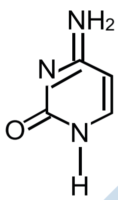
Though the bond order of all the species are same (B.O = 1) but stability is different. This is due to difference in the presence of no. of anti-bonding electron.

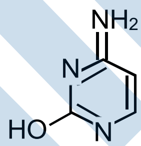
Higher the no. of anti-bonding electron lower is the stability hence the correct order is

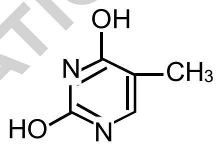
$H_2 > Li_2 > B_2$

47. Which of the following structures represents thymine ?

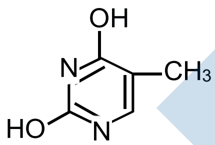
(A) 

(B) 

(C) 

(D*) 

Ans. The correct structure of thymine is



Thymine (T)

48. Which one of the following statements about packing in solids is incorrect ?

(A) Coordination number in bcc mode of packing is 8

(B) Coordination number in hcp mode of packing is 12

(C*) Void space in hcp mode of packing is 32%

(D) Void space in ccp mode of packing is 26%

Ans. The hcp arrangement of atoms occupies 74% of the available space and thus has 26% vacant space.

49. NaOH is a strong base. What will be pH of 5.0×10^{-2} M NaOH solution ? (log2 = 0.3)

(A) 14.00 (B) 13.70 (C) 13.00 (D*) 12.70

Ans. Given $[OH^-] = 5 \times 10^{-2}$

$\therefore$ pOH = – log 5 × 10^{-2}

= – log 5 + 2 log 10 = 1.30

$\because$ pH + pOH = 14

$\because$ pH = 14 – pOH

$= 14 - 1.30 = 12.70$

50. In Goldschmidt alumino thermic process which of the following reducing agents is used :

(A*) Al-powder (B) sodium (C) coke (D) calcium

Ans. Reduction by powdered aluminum is known as Gold-Schmidt aluminothermic process. This process is employed in cases where metals have very high m.p. and are to be extracted from their oxides.

51. Which of the following statements about aspirin is not true ?

(A) It is effective in relieving pain (B*) It belongs to narcotic analgesics

(C) It has antiblood clotting action (D) It is a neurologically active drug

Ans. Aspirin is a non-narcotics analgesic.

52. The polymer used for optical lenses is :

(A*) Polymethyl methacrylate (B) Polythene

(C) Polyvinyl chloride (D) polypropylene

Ans. Polymethyl methacrylate is hard, fairly rigid. It is used for optical lenses.

53. The order of increasing sizes of atomic radii among the elements O, S, Se and As is :

(A) Se < S < As < O (B) As < S < O < Se

(C) O < S < Se < As (D*) O < S < As < Se

Ans. On moving down in a group atomic radii increases due to successive addition of extra shell hence

O < S < Se

Further As is in group 15 having one less electron in its orbital hence have higher atomic radii than group 16 elements.

i.e., O < S < Se < As

54. The reaction $X \to Y$ is an exothermic reaction. Activation energy of the reaction for X into Y is 150 kJ mol^{-1}. Enthalpy of reaction is 135 kJ mol^{-1}. The activation energy for the reverse reaction, $Y \to X$ will be:

(A) 270 kJ mol^{-1} (B) 15 kJ mol^{-1} (C*) 285 kJ mol^{-1} (D) 280 kJ mol^{-1}

Ans. $X \longrightarrow Y$; $\Delta H = -135$ kJ/mol,

$E_a = 150$ kJ/mol

For an exothermic reaction

$E_{a(F.R.)} = \Delta H + E'_{a(B.R.)}$

$150 = -135 + E'_{a(B.R.)}$

$E'_{a(B.R.)} = 285$ kJ/mol

55. Given

Reaction	Energy Change (in kJ)
$Li(s) \to Li(g)$	161
$Li(g) \to Li^+(g)$	520
$\frac{1}{2}F_2(g) \to F(g)$	77

$F(g) + e^- \rightarrow F^-(g)$	(Electron gain enthalpy)
$Li^+(g) + F^-(g) \rightarrow LiF(s)$	–1047
$Li(s) + \frac{1}{2}F_2(g) \rightarrow LiF(s)$	–617

Based on data provided, the value of electron gain enthalpy of fluorine would be :

(A) –300 kJ mol–1 (B) –228 kJ mol^{-1} (C*) – 328 kJ mol^{-1} (D) –350 kJ mol^{-1}

Ans. Applying Hess's Law

$$\Delta_f H^\circ = \Delta_{sub}H + \frac{1}{2}\Delta_{diss}H + I.E. + E.A. + \Delta_{lattice}H$$

– 617 = 161 + 520 + 77 + E.A. + (–1047)

E.A. = –617 + 289 = – 328 kJ mol^{-1}

$\therefore$ electron affinity of fluorine

= –328 kJ mol^{-1}

56. The wave number of the first emission line in the Balmer series of H–Spectrum is :

(R = Rydberg constant) :

(A) $\frac{9}{400}R$ (B) $\frac{3}{4}R$ (C) $\frac{7}{6}R$ (D*) $\frac{5}{36}R$

Ans. $\bar{\nu} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$= R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$

57. For which of the following compounds Kjeldahl method can be used to determine the percentage of Nitrogen ?

(A) Pyridine (B) Nitrobenzene (C*) Alanine (D) Diazomethane

Ans. Kjeldahl's method is not applicable for compounds containing nitrogen in nitro and azo group and nitrogen present in the ring. Because nitrogen of these compounds does not change to ammonium sulphate under these conditions.

Hence only Alanine can be used to determine percentage of nitrogen.

$H_2N – CH – COOH$
(with $CH(CH_3)_2$ attached to CH)

Alanine

58. Amongst the following alcohols which would react fastest with conc. HCl and $ZnCl_2$?

(A*) 2–methylbutan-2-ol (B) 2-pentanol (C) pentanol (D) 2-methylbutanol

Ans. $ZnCl_2$ + conc. HCl is Lucas reagent. Lucas reagent reacts fastest with tertiary alcohol.

3° alcohol + Lucas reagent = Immediate turbidity

59. Flocculation value of $BaCl_2$ is much less than that of KCl for sol A and flocculation value of Na_2SO_4 is much less than that of NaBr for sol B. The correct statement among the following is :

(A) Sol A is negatively charged and sol B is positively charged

(B) Both the sols A and B are positively charged

(C) Both the sols A and B are negatively charged

(D*) Sol A is positively charged and so B is negatively charged.

Ans. In first case the given compounds have same anion but different cations having different charge hence they will precipitate negatively charged sol i.e. 'A'.

60. The density of 3M solution of sodium chloride is 1.252 g mL^{-1}. The molality of the solution will be :

(Molar mass, NaCl = 58.5 g mol^{-1})

(A) 2.18 m (B) 3.00 m (C) 2.60 m (D*) 2.79 m

Ans. The relation between molarity (M) and molality (m) is

$d = M\left(\frac{1}{m} + \frac{M_2}{1000}\right)$, M_2 = Mol. Mass of solute

On putting value

$1.252 = 3\left(\frac{1}{m} + \frac{58.5}{1000}\right)$

on solving m = 2.79

PART-C-MATHEMATICS

61. If a complex number z satisfies the equation $z + \sqrt{2}\ |z + 1| + i = 0$, then | z | is equal to :

(A*) $\sqrt{5}$ (B) 1 (C) 2 (D) $\sqrt{3}$

Ans. Given equation is

$$z+\sqrt{2}\,|z+1|+i=0$$

Put z = x +iy in the given equation.

$$(x+iy)+\sqrt{2}\,|x+iy+1|+i=0$$

$$\Rightarrow x+iy+\sqrt{2}\left[\sqrt{(x+1)^2+y^2}\right]+i=0$$

Now, equating real and imaginary part, we get

$x+\sqrt{2}\sqrt{(x+1)^2+y^2}=0$ and

$y + 1 = 0 \Rightarrow y = -1$

$\Rightarrow x+\sqrt{2}\sqrt{(x+1)^2+(-1)^2}=0$ $(\because y = -1)$

$\Rightarrow \sqrt{2}\sqrt{(x+1)^2+1}=-x$

$\Rightarrow 2[(x + 1)^2 + 1] = x^2$

$\Rightarrow x^2 + 4x + 4 = 0$

$\Rightarrow x = -2$

Thus, $z = -2 + i(-1) \Rightarrow |z| = \sqrt{5}$

62. Given two independent events, if the probability that exactly one of them occurs is $\frac{26}{49}$ and the probability that none of them occurs is $\frac{15}{49}$, then the probability of more probable of the two events is :

(A) $\frac{5}{7}$ (B) $\frac{3}{7}$ (C*) $\frac{4}{7}$ (D) $\frac{6}{7}$

Ans. Let the probability of occurrence of first event A, be 'a'

i.e., P(A) = a

$\therefore$ P(not A) = 1 – a

And also suppose that probability of occurrence of second event B, P(B) = b,

$\therefore$ P(not B) = 1 – b

Now, P(A and not B) + P(not A and B) $= \frac{26}{49}$

$$\Rightarrow P(A)\times P(\text{not } B)+P(\text{not } A)\times P(B)=\frac{26}{49}$$

$\Rightarrow a \times (1-b) + (1-a)b = \frac{26}{49}$

$\Rightarrow a + b - 2ab = \frac{26}{49}$ ......(i)

And $P(\text{not A and not B}) = \frac{15}{49}$

$\Rightarrow P(\text{not A}) \times P(\text{not B}) = \frac{15}{49}$

$\Rightarrow (1-a) \times (1-b) = \frac{15}{49}$

$\Rightarrow 1 - b - a + ab = \frac{15}{49}$

$\Rightarrow a + b - ab = \frac{34}{49}$ ......(ii)

From (i) and (ii),

$a + b = \frac{42}{49}$ ......(iii)

and $ab = \frac{8}{49}$

$(a-b)^2 = (a+b)^2 - 4ab = \frac{42}{49} \times \frac{42}{49} \times \frac{4 \times 8}{49} = \frac{196}{2401}$

$\therefore a - b = \frac{14}{49}$ ......(iv)

From (iii) and (iv),

$a = \frac{4}{7}, b = \frac{2}{7}$

Hence probability of more probable of the two events = $\frac{4}{7}$

63. Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing then the variance of all the five observations is :

(A) 6 (B) 8 (C*) 4 (D) 2

Ans. Let 5th observation be x.

Given mean = 7

$\therefore 7 = \frac{6+7+8+10+x}{5}$

$\Rightarrow x = 4$

Now, Variance

$= \sqrt{\frac{(6-7)^2 + (7-7)^2 + (8-7)^2 + (10-7)^2 + (4-7)^2}{5}}$

$$= \sqrt{\frac{1^2+0^2+1^2+3^2+3^2}{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$$

64. The number of solutions of the equation $\sin^{-1}x = 2\tan^{-1}x$ (in principal values) is :

(A) 4 (B) 3 (C*) 1 (D) 2

Ans. Given equation is

$\sin^{-1} x = 2 \tan^{-1} x$

Now, this equation has only one solution.

$\therefore$ LHS $= \sin^{-1}1 = \frac{\pi}{2}$

and RHS $= 2\tan^{-1}1 = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

Also, x = 1 gives angle value as $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

$\frac{5\pi}{4}$ is outside the principal value.

65. If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$, such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set :

(A*) {– 2, 5} (B) {3, – 5} (C) {2, – 5} (D) {– 3, 2}

Ans. Given quadratic equation is

$x^2 + px + \frac{3p}{4} = 0$

So, $\alpha + \beta = -p$, $\alpha\beta = \frac{3p}{4}$

Now, given $|\alpha - \beta| = \sqrt{10}$

$\Rightarrow \alpha - \beta = \pm\sqrt{10}$

$\Rightarrow (\alpha - \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - \alpha\beta = 10$

$\Rightarrow (\alpha - \beta)^2 - 4\alpha\beta = 10$

$\Rightarrow p^2 - 4 \times \frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$

$\Rightarrow p = -2, 5 \Rightarrow p \in \{-2, 5\}$

66. The acute angle between two lines such that the direction cosines 1, m, n of each of them satisfy the equations $1 + m + n = 0$ and $1^2 + m^2 + n^2 = 0$ is :

(A*) 60° (B) 30° (C) 15° (D) 45°

Ans. Let ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 be the d.c of line

1 and 2 respectively, then as given

and $\ell_2 + m_2 + n_2 = 0$

and $\ell_1^2 + m_1^2 - n_1^2 = 0$ and

$\ell_2^2 + m_2^2 - n_2^2 = 0$

$(\because \ell + m + n = 0 \text{ and } \ell^2 + m^2 + n^2 = 0)$

Angle between lines, θ is

$\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$ ....(1)

As given $\ell^2 + m^2 = n^2$ and $\ell + m = -n$

$\Rightarrow (-n)^2 - 2\ell m = n^2 \Rightarrow 2\ell m = 0$ or $\ell m = 0$

So $\ell_1 m_1 = 0$, $\ell_2 m_2 = 0$

If $\ell_1 = 0$, $m_1 \neq 0$ then $\ell_1 m_2 = 0$

If $m_1 = 0$, $\ell_1 \neq 0$ then $\ell_2 m_1 = 0$

If $\ell_2 = 0$, $m_2 \neq 0$ then $\ell_2 m_1 = 0$

If $m_2 = 0$, $\ell_2 \neq 0$ then $\ell_1 m_2 = 0$

Also $\ell_1\ell_2 = 0$ and $m_1m_2 = 0$

$\ell^2 + m^2 - n^2 = \ell^2 + m^2 + n^2 - 2n^2 = 0$

$\Rightarrow 1 - 2n^2 - 0 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore n_1 = \pm\frac{1}{\sqrt{2}}, n_2 = \pm\frac{1}{\sqrt{2}}$

$\therefore \cos\theta = \frac{1}{2}$ $\theta = 60°$ (acute angle)

67. Let the equations of two ellipses be

$E_1 : \frac{x^2}{3} + \frac{y^2}{2} = 1$ and $E_2 : \frac{x^2}{16} + \frac{y^2}{b^2} = 1.$

If the product of their eccentricities is $\frac{1}{2}$, then the length of the minor axis of ellipse E_2 is :

(A) 9 (B*) 4 (C) 8 (D) 2

Ans. Given equation of ellipses

$E_1 : \frac{x^2}{3} + \frac{y^2}{2} = 1$

$\Rightarrow e_1 = \sqrt{1 - \frac{2}{3}} = \frac{1}{\sqrt{3}}$

and $E_2 : \frac{x^2}{16} + \frac{y^2}{b^2} = 1$

$\Rightarrow e_2 = \sqrt{\frac{1 - b^2}{16}} = \sqrt{\frac{16 - b^2}{4}}$

Also, given $e_1 \times e_2 = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{3}} \times \sqrt{\frac{16-b^2}{4}} = \frac{1}{2} \Rightarrow 16 - b^2 = 12$

$\Rightarrow b^2 = 4$

$\therefore$ Length of minor axis of

$E_2 = 2b = 2 \times 2 = 4$

68. The number of ways in which an examiner can assign 30 marks to 8 questions, giving not less than 2 marks to any question, is :

(A*) ${}^{21}C_7$ (B) ${}^{21}C_8$ (C) ${}^{30}C_7$ (D) ${}^{30}C_8$

Ans. 30 marks to be allotted to 8 questions. Each question has to be given ≥ 2 marks
Let questions be a, b, c, d, e, f, g, h
and a + b + c + d + e + f + g + h = 30
Let $a = a_1 + 2$ so, $a_2 \geq 0, \ldots\ldots, a_8 \geq 0$

So, $\left.\begin{matrix} a_1 + a_2 + \ldots\ldots + a_8 \\ +2+2+\ldots\ldots+2 \end{matrix}\right\} = 30$

$\Rightarrow a_1 + a_2 + \ldots.. + a_8 = 30 - 16 = 14$
So, this is a problem of distributing 14 articles in 8 groups.
Number of ways = ${}^{14+8-1}C_{8-1} = {}^{21}C_7$

69. **Statement-1:** The line x – 2y = 2 meets the parabola, $y^2 + 2x = 0$ only at the point (– 2, – 2).

Statement-2: The line $y = mx - \frac{1}{2m}$ $(m \neq 0)$ is tangent to the parabola, $y^2 = -2x$ at the point $\left(\frac{-1}{2m^2}, \frac{-1}{m}\right)$

(A) Statement-1 is true; Statement-2 is false

(B) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(C*) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(D) Statement-1 is false; Statement-2 is true.

Ans. Both statements are true and statement-2 is the correct explanation for statement-1

$\therefore$ The straight line $y = mx + \frac{a}{m}$ is always a tangent to the parabola $y^2 = 4ax$ for any value of m.

The co-ordinates of point of contact $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

70. If two vertices of an equilateral triangle are A (– a, 0) and B (a, 0), a > 0 and the third vertex C lies above x-axis then the equation of the circumcircle of ΔABC is :

(A*) $3x^2 + 3y^2 - 2\sqrt{3}\,ay = 3a^2$ (B) $3x^2 + 3y^2 - 2ay = 3a^2$

(C) $x^2 + y^2 - \sqrt{3}\,ay = a^2$ (D) $x^2 + y^2 - 2ay = a^2$

Ans. Let C = (x, y)

Now, $CA^2 = CB^2 = AB^2$

$\Rightarrow (x + a)^2 + y^2 = (x - a)^2 + y^2 = (2a)^2$

$\Rightarrow x^2 + 2ax + a^2 + y^2 = 4a^2$ ......(i)

and $x^2 - 2ax + a^2 + y^2 = 4a^2$ ......(ii)

From (i) and (ii), x = 0 and $y = \pm\sqrt{3}a$

Since point C(x, y) lies above the x-axis and a > 0, hence $y = \sqrt{3}a$

$\therefore C = (0\sqrt{3}a)$

Let the equation of circumcircle be

$x^2 + y^2 + 2gx + 2fy + C = 0$

Since points A(–a, 0), B(a, 0) and C $(0, \sqrt{3}a)$ lie on the circle, therefore

$a^2 - 2ga + C = 0$ ......(iii)

$a^2 + 2ga + C = 0$ ......(iv)

and $3a^2 + 2\sqrt{3}af + C = 0$ ......(iii)

From (iii), (iv) and (V)

$g = 0, c = -a^2, f = -\frac{a}{\sqrt{3}}$

Hence equation of the circumcircle is

$$x^2 + y^2 - \frac{2a}{\sqrt{3}}y - a^2 = 0$$

$$\Rightarrow x^2 + y^2 - \frac{2\sqrt{3}ay}{3} - a^2 = 0$$

$$\Rightarrow 3x^2 + 3y^2 - 2\sqrt{3}ay = 3a^2$$

71. Consider the differential equation $\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$.

Statement-1: The substitution z = y^2 transforms the above equation into a first order homogenous differential equation.

Statement-2: The solution of this differential equation is $y^2 e^{\frac{-y^2}{x}} = C$.

(A) both statements are false.

(B) Statement-1 is false; Statement-2 is true then

(C) Statement-1 is true; Statement-2 is false

(D*) both statements are true.

Ans. Given differential equation is

$$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$$

By substituting $z = y^2$, we get diff. eqn. as

$$\frac{dz}{dx} = \frac{2z^3}{2(xz - x^2)} = \frac{z^2}{xz - x^2}$$

Now, $\frac{dx}{dz} = \frac{x}{z} - \frac{x^2}{z^2} = \frac{x}{z}\left[1 - \frac{x}{z}\right] \approx F\left(\frac{x}{z}\right)$

Hence, statement-1 is true.

Now, $y^2 e^{-y^2/x} = C$ satisfies the given diff. equation

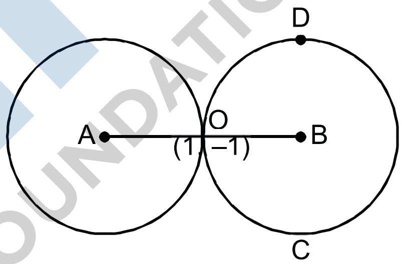
$\therefore$ It is the solution of given diff. equation.

Thus, statement-2 is also true.

72. If a circle C passing through (4, 0) touches the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ externally at a point (1, – 1), then the radius of the circle C is :

(A) $\sqrt{57}$ (B) 4 (C) $2\sqrt{5}$ (D*) 5

Ans. Let A be the centre of given circle and B be the centre of circle C.



$x^2 + y^2 + 4x - 6y - 12 = 0$

$\therefore$ A = (–2, 3) and B = (g, f)

Now, from the figure, we have

$\frac{-2+g}{2} = 1$ and $\frac{3+f}{2} = -1$ (By mid-point formula)

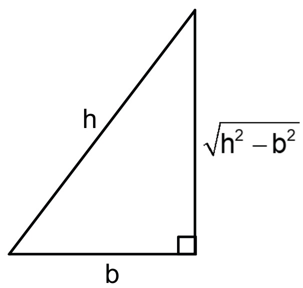
$\Rightarrow$ g = 4 and f = –5

Now, required radius = OB $= \sqrt{9+16} = \sqrt{25} = 5$

73. The maximum area of a right angled triangle with hypotenuse h is :

(A) $\frac{h^2}{2}$ (B*) $\frac{h^2}{4}$ (C) $\frac{h^2}{\sqrt{2}}$ (D) $\frac{h^2}{2\sqrt{2}}$

Ans. Let base = b



Altitude (or perpendicular) $= \sqrt{h^2 - b^2}$

Area, $A = \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times b \times \sqrt{h^2 - b^2}$

$\Rightarrow \frac{dA}{db} = \frac{1}{2}\left[\sqrt{h^2 - b^2} + b \cdot \frac{-2b}{2\sqrt{h^2 - b^2}}\right]$

$= \frac{1}{2}\left[\frac{h^2 - 2b^2}{\sqrt{h^2 - b^2}}\right]$

Put $\frac{dA}{db} = 0, \Rightarrow b = \frac{h}{\sqrt{2}}$

Maximum area $= \frac{1}{2} \times \frac{h}{\sqrt{2}} \times \sqrt{h^2 - \frac{h^2}{2}} = \frac{h^2}{4}$

74. If the 7th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x\right)^9$, x > 0 is equal to 729, then x can be :

(A*) e (B) e^2 (C) 2e (D) $\frac{e}{2}$

Ans. Let r + 1 = 7 ⇒ r = 6

Given expansion is

$\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x\right)^9, x > 0$

We have

$T_{r+1} = {}^nC_r\,(x)^{n-r}\,a^r$ for $(x + a)^n$.

∴ According to the question

$729 = {}^9C_6\left(\frac{3}{\sqrt[3]{84}}\right)^3 \cdot (\sqrt{3}\ln x)^6$

$\Rightarrow 3^6 = 84 \times \frac{3^3}{84} \times 3^3 \times (6\ln x)$

$\Rightarrow (\ln x)^6 = 1 \Rightarrow (\ln x)^6 = (\ln e)^6$

$\Rightarrow x = e$

75. If $\int \frac{x^2 - x + 1}{x^2 + 1} e^{\cot^{-1}x}\,dx = A(x)e^{\cot^{-1}x} + C$, then A(x) is equal to :

(A*) x (B) – x (C) $\sqrt{1-x}$ (D) $\sqrt{1+x}$

Ans. Let $I = \int \frac{x^2 - x + 1}{x^2 + 1} \cdot e^{\cot^{-1}x}\,dx$

Put x = cot t ⇒ – cosec^2 t dt = dx

Now, 1 + $\cot^2$ t = cosec^2 t

$\therefore I = \int \frac{e^t(\cot^2 t - \cot t + 1)}{(1 + \cot^2 t)}(-\text{cosec}^2 t)dt$

$= -\int e^t(\text{cosec}^2 t - \cot t)dt$

$= \int e^t(\cot t - \text{cosec}^2 t)dt$

$= e^t \cot t + C$

$= e^{\cot^{-1}x}(x) + C \equiv A(x).e^{\cot^{-1}x} + C$

⇒ A(x) = x

76. The area of the region (in sq. units), in the first quadrant, bounded by the parabola $y = 9x^2$ and the lines x = 0, y = 1 and y = 4 is :

(A) $\frac{14}{3}$ (B*) $\frac{14}{9}$ (C) $\frac{7}{3}$ (D) $\frac{7}{9}$

Ans. Required area $= \int_{y=1}^{4} \sqrt{\frac{y}{9}}dy$

$$= \frac{1}{3}\int_{y=1}^{4} y^{1/2}dy = \frac{1}{3} \times \frac{2}{3}(y^{3/2})\Big|_1^4$$

$$= \frac{2}{9}[(4^{1/2})^3 - (1^{1/2})^3] = \frac{2}{9}[8-1]$$

$$= \frac{2}{9} \times 7 = \frac{14}{9} \text{sq. units.}$$

77. The statement $p \to (q \to p)$ is equivalent to

(A) $p \to q$ (B) $p \to (p \to q)$ (C*) $p \to (p \vee q)$ (D) $p \to (p \wedge q)$

Ans.

q	p	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Since truth value of $p \to (q \to p)$ and $p \to (p \vee q)$ are same, hence $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

78. **Statement-1:** The function $x^2 (e^x + e^{-x})$ is increasing for all $x > 0$.

Statement-2: The functions x^2e^x and x^2e^{-x} are increasing for all $x > 0$ and the sum of two increasing functions in any interval (a, b) is an increasing function in (a, b).

(A) Statement-1 is false; Statement-2 is true.

(B*) Statement-1 is true; Statement-2 is false

(C) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(D) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

Ans. Let $y = x^2.e^{-x}$

For increasing function,

$$\frac{dy}{dx} > 0 \Rightarrow x[(2-x)e^{-x}] > 0$$

$\because x > 0, \therefore (2-x)e^{-x} > 0$

$$\Rightarrow (2-x)\frac{1}{e^x} > 0$$

For $0 < x < 2$, $(2-x) < 0$

$\therefore \frac{1}{e^x} < 0$, but it is not possible

Hence the statement-2 is false.

79. For $a > 0, t \in \left(0, \frac{\pi}{2}\right)$, let $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$. Then $1 + \left(\frac{dy}{dx}\right)^2$ equals :

(A*) $\frac{x^2+y^2}{x^2}$ (B) $\frac{x^2}{y^2}$ (C) $\frac{y^2}{x^2}$ (D) $\frac{x^2+y^2}{y^2}$

Ans. Let $x = \sqrt{a^{\sin^{-1}t}}$

$\Rightarrow x^2 = a^{\sin^{-1}t} \Rightarrow 2\log x = \sin^{-1}t.\log a$

$\Rightarrow \frac{2}{x} = \frac{\log a}{\sqrt{1-t^2}} \cdot \frac{dt}{dx}$

$\Rightarrow \frac{2\sqrt{1-t^2}}{x\log a} = \frac{dt}{dx}$ ......(1)

Now, let $y = \sqrt{a^{\cos^{-1}t}}$

$\Rightarrow 2 \log y = \cos^{-1} t. \log a$

$\Rightarrow \frac{2}{y} \cdot \frac{dy}{dx} = \frac{-\log a}{\sqrt{1-t^2}} \cdot \frac{dt}{dx}$

$\Rightarrow \frac{2}{y} \cdot \frac{dy}{dx} = \frac{-\log a}{\sqrt{1-t^2}} \times \frac{2\sqrt{1-t^2}}{x\log a}$ form (1)

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Hence, $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{-y}{x}\right)^2 = \frac{x^2+y^2}{x^2}$

80. Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is :

(A) $3n + 2n^2$ (B*) $6n^2 - n$ (C) $n + 4n^2$ (D) $n^2 + 4n$

Ans. Given $S_n = 2n + 3n^2$

Now, first term = 2 + 3 = 5

second term = 2(2) + 3(4) = 16

third term = 2(3) + 3(9) = 33

Now, sum given in option (b) only has the same first term and difference between 2nd and 1st term is double also.

81. If the system of linear equations

$x_1 + 2x_2 + 3x_3 = 6$

$x_1 + 3x_2 + 5x_3 = 9$

$2x_1 + 5x_2 + ax_3 = b$

is consistent and has infinite number of solutions, then :

(A) b = 15, a can be any real number

(B*) a = 8, b =15

(C) $a \in R - \{8\}$ and $b \in R - \{15\}$

(D) a = 8, b can be any real number

Ans. Given system of equations can be written in matrix form as AX = B where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix}$$

Since, system is consistent and has infinitely many solutions

$\therefore$ (adj. A) B = 0

$$\Rightarrow \begin{pmatrix} 3a-25 & 15-2a & 1 \\ 10-a & a-6 & -2 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow -6 - 9 + b = 0 \Rightarrow b = 15$

and $6(10 - a) + 9(a - 6) - 2(b) = 0$

$\Rightarrow 60 - 6a + 9a - 54 - 30 = 0$

$\Rightarrow 3a = 24 \Rightarrow a = 8$

Hence, a = 8, b = 15.

82. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors satisfying $\hat{a} - \sqrt{3}\,\hat{b} + \hat{c} = \vec{0}$, then the angle between the vectors $\hat{a}$ and $\hat{c}$ is:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C*) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Ans. Let angle between $\hat{a}$ and $\hat{c}$ be θ.

Now, $\hat{a} - \sqrt{3}\hat{b} + \hat{c} = \vec{0}$

$\Rightarrow (\hat{a} + \hat{c}) = \sqrt{3}\hat{b}$

$\Rightarrow (\hat{a} + \hat{c}).(\hat{a} + \hat{c}) = 3(\hat{b}.\hat{b})$

$\Rightarrow \hat{a}.\hat{a} + \hat{a}.\hat{c} + \hat{c}.\hat{a} + \hat{c}.\hat{c} = 3 \times 1$

$\Rightarrow 1 + 2\cos\theta + 1 = 3$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

83. If p, q, r are 3 real numbers satisfying the matrix equation, $\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [3 \;\; 0 \;\; 1]$, then

(2p + q – r) equals :

(A*) – 3 (B) 2 (C) 4 (D) – 1

Ans. Given

$$[p \quad q \quad r]\begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = [3 \quad 0 \quad 1]$$

$\Rightarrow [3p + 3q + 2r \quad 4p + 2q \quad p + 3q + 2r] = [3 \quad 0 \quad 1]$

$\Rightarrow 3p + 3q + 2r = 3$ ....(i)

$4p + 2q = 0 \Rightarrow q = -2p$ .....(ii)

$p + 3q + 2r = 1$ .....(iii)

on solving (i), (ii) and (iii) we get

p = 1, q = –2, r = 3

$\therefore$ 2p + q – r = 2(1) + (–2) – (3) = –3.

84. If the x-intercept of some line L is double as that of the line, 3x + 4y = 12 and the y-intercept of L is half as that of the same line, then the slope of L is :

(A) $\frac{-3}{8}$ (B) $\frac{-3}{2}$ (C*) $\frac{-3}{16}$ (D) – 3

Ans. Given line 3x + 4y = 12 can be rewritten as

$$\frac{3x}{12} + \frac{4y}{12} = 1 \Rightarrow \frac{x}{4} + \frac{y}{3} = 1$$

$\Rightarrow$ x-intercept = 4 and y-intercept = 3

Let the required line be

$L : \frac{x}{a} + \frac{y}{b} = 1$ where

a = 4 × 2 = 8 and b = 3/2

$\therefore$ Required line is $\frac{x}{8} + \frac{2y}{3} = 1$

$\Rightarrow$ 3x + 16y = 24

$\Rightarrow y = \frac{-3}{16}x + \frac{24}{16}$

Hence, required slope $= \frac{-3}{16}$

85. Let R = {(3, 3), (5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5)} be a relation on the set A = {3, 5, 9, 12}. Then R is :

(A*) reflexive, transitive but not symmetric (B) an equivalence relation

(C) reflexive, symmetric but not transitive (D) symmetric, transitive but not reflexive

Ans. Let R = {(3, 3), (5, 5), (9, 9), (12, 12), (5, 12), (3, 9), (3, 12), (3, 5)} be a relation on the set A = {3, 5, 9, 12}

Clearly, every element of A is related to itself.

Therefore, it is a reflexive.

Now, R is not summetry because 3 is related to 5 but 5 is not related to 3.

Also R is transitive relation because it satisfies the property that if a R b and b R c then a R c.

86. **Statement-1:** The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two.

Statement-2: The number of solutions of the equations, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two.

(A) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1

(B*) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1

(C) Statement-1 is false; Statement-2 is true.

(D) Statement-1 is true; Statement-2 is false.

Ans. $2\sin^2\theta - \cos 2\theta = 0$

$\Rightarrow 2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$

$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Now $2\cos^2\theta - 3\sin\theta = 0$

$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$

$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$

$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$

$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}, -2$

But $\sin\theta = -2$, is not possible

$\therefore \sin\theta = \frac{1}{2}, \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Hence, there are two common solution, there each of the statement-1 and 2 are true but statement-2 is not a correct explanation for statement-1.

87. Let $f(x) = -1 + |x - 2|$, and $g(x) = 1 - |x|$; then the set of all points where fog is discontinuous is

(A*) an empty set (B) {0, 1, 2} (C) {0, 2} (D) {0}

Ans. $fog = f(g(x)) = f(1 - |x|)$

$= -1 + |1 - |x| - 2|$

$= -1 + |-|x| - 1| = -1 + ||x| + 1|$

Let fog = y

$\therefore$ y = – 1 + ||x| + 1|

$$\Rightarrow y = \begin{cases} -1+x+1, & x \ge 0 \\ -1-x+1, & x < 0 \end{cases}$$

$$\Rightarrow y = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$$

LHL at (x = 0) = $\lim_{x\to 0}(-x) = 0$

RHL at (x = 0) = $\lim_{x\to 0}(-x) = 0$

When, x = 0, then y = 0

Hence, LHL at (x = 0) = RHL at (x = 0) = value of y at (x = 0)

Hence y is continuous at x = 0.

Clearly at all other point y continuous.

Therefore, the set of all points where fog is discontinuous is an empty set.

88. The sum $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + ...n$ upto 11-terms is :

(A) $\frac{60}{11}$ (B) $\frac{11}{4}$ (C*) $\frac{11}{2}$ (D) $\frac{7}{2}$

Ans. Given sum is

$$\frac{3}{12} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + ...$$

nth term = T_n

$$= \frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$$

$$\text{or } T_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

$$\therefore S_n = \sum T_n = 6\sum \frac{1}{n} - 6\sum \frac{1}{n+1} = \frac{6}{n} - \frac{6}{n+1}$$

$$= 6 - \frac{6}{n+1} = \frac{6n}{n+1}$$

So, sum upto 11 terms means

$$S_{11} = \frac{6\times 11}{11+1} = \frac{66}{12} = \frac{33}{6} = \frac{11}{2}$$

89. Let Q be the foot of perpendicular from the origin to the plane 4x – 3y + z + 13 = 0 and R be a point (– 1, 1, – 6) on the plane. Then length QR is :

(A*) $3\sqrt{\frac{7}{2}}$ (B) $\sqrt{14}$ (C) $\sqrt{\frac{19}{2}}$ (D) $\frac{3}{\sqrt{2}}$

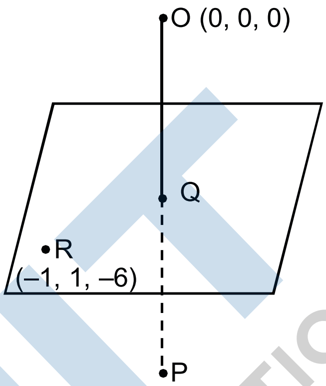
Ans. Let P be the image of O in the given plane.

Equation of the plane, $4x - 3y + z + 13 = 0$ Op is normal to the plane, therefore direction ratio of OP are proportional to 4, –3, 1

Since OP passes through (0, 0, 0) and has direction ration proportional to 4, –3, 1.

Therefore equation of OP is

$$\frac{x-0}{4} = \frac{y-0}{-3} = \frac{z-0}{1} = r \text{ (let)}$$



$\therefore x = 4r,\ y = -3r,\ z = r$

Let the coordinate of P be (4r, –3r, r)

Since Q be the mind point of OP

$$\therefore Q = \left(2r, -\frac{3}{2}r, \frac{r}{2}\right)$$

Since Q lies in the given plane

$4x - 3y + z + 13 = 0$

$$\therefore 8r + \frac{9}{2}r + \frac{r}{2} + 13 = 0$$

$$\Rightarrow r = \frac{-13}{8 + \frac{9}{2} + \frac{1}{2}} = \frac{-26}{26} = -1$$

$$\therefore Q = \left(-2, \frac{3}{2}, -\frac{1}{2}\right)$$

$$\therefore QR = \sqrt{(-1+2)^2 + \left(1 - \frac{3}{2}\right)^2 + \left(-6 + \frac{1}{2}\right)^2}$$

$$= \sqrt{1 + \frac{1}{4} + \frac{121}{4}} = 3\sqrt{\frac{7}{2}}$$

90. The integral $\int_{\frac{7\pi}{4}}^{\frac{7\pi}{3}} \sqrt{\tan^2 x}\, dx$ is equal to :

(A) log 2 (B) 2log 2 (C*) $\log\sqrt{2}$ (D) $\log 2\sqrt{2}$

Ans. Let $I = \int_{7\pi/4}^{7\pi/3} \sqrt{\tan^2 x}\, dx$

$$= \int_{7\pi/4}^{7\pi/3} \tan x\, dx = -\log \cos x \Big|_{7\pi/4}^{7\pi/3}$$

$$= -\left[\log \cos\frac{7\pi}{3} - \log \cos\frac{7\pi}{4}\right]$$

$$= \log \cos\frac{7\pi}{4} - \log \cos\frac{7\pi}{3}$$

$$= \log\left[\frac{\cos\frac{7\pi}{4}}{\cos\frac{7\pi}{3}}\right] = \log\left[\frac{\cos\left(2\pi - \frac{\pi}{4}\right)}{\cos\left(2\pi + \frac{\pi}{3}\right)}\right]$$

$$= \log\left(\frac{\cos\frac{\pi}{4}}{\cos\frac{\pi}{3}}\right) = \log\left(\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\right)$$

$$= \log\left(\frac{2}{\sqrt{2}}\right) = \log\sqrt{2}.$$